26 $f(t)=\frac{2}{1-t^2}$ $\quad g(x)=\int_0^x f(t)\,dt$

a) geo. $a=2$

$r=t^2$

$\frac{a}{1-r}$

series: $f(t)=2+2t^2+2t^4+2t^6\ldots$

general term $2t^{2n}$

b) $g(t)=2t+\frac{2t^3}{3}+\frac{2t^5}{5}+\frac{2t^7}{7}+\ldots\Big|_0^x$

$g(x)=2x+\frac{2x^3}{3}+\frac{2x^5}{5}+\frac{2x^7}{7}+\ldots\frac{2x^{2n+1}}{2n+1}$

n starts at 0

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43. $f(x)=\frac{\sin x}{x}$ $\quad \lim_{x\to 0}\frac{\sin x}{x}=1$

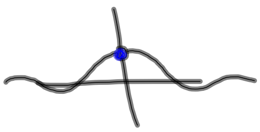
a) $\frac{\sin x}{x}=\frac{\cancel{x}}{\cancel{x}}-\frac{x^3}{\cancel{x}\cdot 3!}+\frac{x^5}{\cancel{x}\cdot 5!}-\frac{x^7}{\cancel{x}\cdot 7!}\ldots$

$=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}\ldots+\frac{(-1)^n x^{2n}}{(2n+1)!}$

b) undefined at $x=0$ $\frac{\sin x}{x}$

n starts at 0

c) $f(x)=\begin{cases}\frac{\sin x}{x} & \text{if } x\neq 0\\ 1 & x=0\end{cases}$



d) $\int_0^x \frac{\sin t}{t}\,dt = t-\frac{t^3}{3\cdot 3!}+\frac{t^5}{5\cdot 5!}-\frac{t^7}{7\cdot 7!}\ldots\Big|_0^x$ $\quad f(t)$

$=x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\frac{x^7}{7\cdot 7!}\ldots$

$\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$

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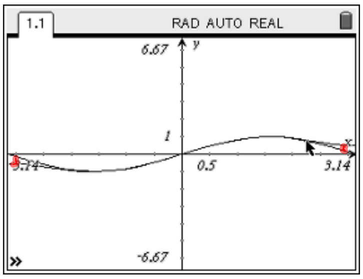
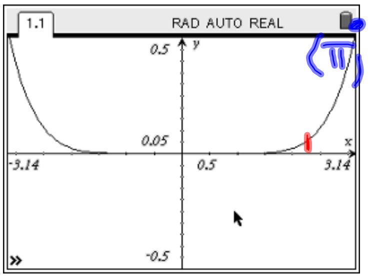
9.3 Taylor Series Remainder Theorem

$$\overset{f_1(x)}{\sin x} \approx \overset{f_2(x)}{x - \frac{x^3}{3!} + \frac{x^5}{5}} \quad \text{on } [-\pi, \pi]$$



$$\text{error} = \overset{f_3(x)}{\left| f_1(x) - f_2(x) \right|}$$

what is the max error on $[-\pi, \pi]$? $= .5240$



f3

$(\pi, .5240)$

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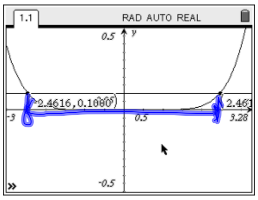
On what interval is the max error .1?

$[-2.4616, 2.4616]$



$f_4(x) = .1$

Estimate the error:

Taylor's Remainder Estimation Theorem (Lagrange)

$$f(x) \approx f(a) + f'(a)(x-a) + \dots + \frac{f^n(a)}{n!}(x-a)^n$$

Stop here

$$f(x) \approx p(x)$$

$$\text{error} = \left| f(x) - p(x) \right|$$

$$\boxed{\text{error} \approx \frac{M(x-a)^{n+1}}{(n+1)!} \quad \text{where } M \geq f^{n+1}(x)}$$

looks kind of like next term

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$$\sin x \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

exact error $= \left| \sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)\right|$

$f1(x) - f2(x)$

Taylor's Remainder Thm

$$\text{error} \le \left| M \frac{x^7}{7!} \right|$$

$M \ge 7^{th}$ derivative of $\sin(x)$

$M = 1$

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How many terms are needed so the max error is .05 on $[-\pi, \pi]$. (to approx $\sin(x)$)

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots \frac{x^{2n+1}}{(2n+1)!}(-1)^n$$

$$\text{error} \le \left| \frac{M \cdot x^{2n+3}}{(2n+3)!} \right| < .05$$

$M = 1$

next term

let $x = \pi$

$$\left| \frac{\pi^{2n+3}}{(2n+3)!} \right| < .05$$

$n = 4$ best

$n = 5$

trial & error

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